Enrollment No. ____________

Shree Manibhai Virani and Smt. Navalben Virani Science College (Autonomous), Rajkot
Affiliated to Saurashtra University, Rajkot

SEMESTER END EXAMINATION NOVEMBER - 2017

M.Sc. Mathematics

16PMTCC14 – DISCRETE MATHEMATICS

Duration of Exam – 3 hrs ***Semester – III*** ***Max. Marks – 70***

***Part A* (5X2= 10 marks)**

Answer **ALL** questions

1. Let S be a non empty set and $\wp(S)$ be its power set. Find identity and zero elements of $(\wp(S),\cap)$.
2. Find minimal and maximal elements of poset (P, D) with $P = \{2,4,5,6,,8,10,12,16\}$ and $D = Divisibility\ relation$.
3. Define Non deterministic finite automata with $\wedge$-transitions.
4. Define predicate with example.
5. Define the terms: Channel, Decoder

***Part B* (5X5= 25 marks)**

Answer **ALL** questions

6a. Define congruence relation on semigroups. Show that if $(S,*)$ and (T, Δ) be two semigroups and g be a semigroup homomorphism from $(S,*)$ to (T, Δ). Corresponding to homomorphism g, there exists a congruence relation R on $(S,*)$ defined by $xRy \;\; iff \;\; g(x) = g(y) \;\; for\ x, y \in S$

OR

6b. Show that set of idempotent elements in a commutative monoid forms a submonoid.

7a. Let $(L, \leq)$ be a lattice. Then show that for any $a, b \in L$ $\mathrm{a} \leq \mathrm{b} \iff \mathrm{a} * \mathrm{b} = \mathrm{a} \iff \mathrm{a} \oplus \mathrm{b} = \mathrm{b}$

OR

7b. State and prove Modular inequality for lattices.

8a. Defined extended transition function δ^* for NFA and then obtain $\delta^*(q_0, 010)$ in following NFA .

q	$\delta(q,0)$	$\delta(q,1)$
q_0	$\{q_0\}$	$\{q_0, q_1\}$
q_1	$\{q_2\}$	$\{q_2\}$
q_2	$\{q_3\}$	$\{q_3\}$
q_3	φ	φ

OR

8b. Draw FAs corresponding to following regular expressions.

i) $(11 + 10)^*$ ii) $(0 + 1)^*(1 + 00)(0 + 1)^*$

9a. Define universal quantifier and existential quantifier with examples.

OR

9b. State and prove generalized DeMorgan's laws in first order logic.

10a. From following parity check matrix obtain generator matrix and using it find all generated codewords. $H = \begin{bmatrix} 1\,1\,1\,0\,1\,0\,0 \\ 1\,1\,0\,1\,0\,1\,0 \\ 1\,0\,1\,1\,0\,0\,1 \end{bmatrix}$

OR

10b. Show that A code C can correct all combination of k or fewer or error if and only if $d(C) \geq k+1$

<u>Part C</u> (5X7= 35 marks)

Answer **<u>ALL</u>** questions

11a. Define Warshall's Algorithm and find transitive closure of following relation $\begin{bmatrix} 0\,1\,0\,1 \\ 1\,0\,1\,0 \\ 1\,0\,0\,0 \\ 0\,0\,0\,0 \end{bmatrix}$

OR

11b. Define free semi group. Let X be the set containing n elements, let X^* denote the free semigroup generated by X, and let $(S, \oplus)$ be any other semigroup generate any n generators; then there exist a homomorphism $g: X^* \to S$.

12a. Let $(B, *, \oplus, ', 0, 1)$ be Boolean algebra. Then show that for any $a, b \in B$
$a = b \Leftrightarrow ab' + a'b = 0$ and $a = 0 \Leftrightarrow ab' + a'b = b$

OR

12b. Show that a lattice $(L, *, \oplus)$ is distributive then for any $a, b, c \in L$
$(a * b) \oplus (b * c) \oplus (c * a) = (a \oplus b) * (b \oplus c) * (c \oplus a)$

13a. Show that for any NFA $M = (Q, \Sigma, q_0, A, \delta)$ accepting a language $L \subseteq \Sigma^*$, there is an FA $M_1 = (Q_1, \Sigma, q_1, A_1, \delta_1)$ that also accepts L.

OR

13b. If L_1 and L_2 are context free languages then show that $L_1 L_2$ is also context free language.
Using above result find context free grammar of language $L = \{0^i 1^j 0^k \mid j > i + k\}$.

14a. Show that following statements are equivalent
p: n is an odd integer.
q: $5n + 4$ is an odd integer.
r: n^2 is an odd integer.

OR

14b. Prove following statements
(i) For all integer n, If $5n + 2$ is an odd integer then n is an odd integer
(ii) Prove by contradiction method $\sqrt{5}$ is an irrational no.

15a. Show that
A binary code C can correct up to k errors in any codeword if and only if
$d(C) \geq 2k + 1$.

OR

15b. Explain error recovery in group code using coset leader with suitable example.